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Signature of short distance physics on inflation power spectrum and CMB anisotropy

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Abstract: The inflaton field responsible for inflation may not be a canonical fundamental scalar. It is possible that the inflaton is a composite of fermions or it may have a decay width. In these cases the standard procedure for calculating the power spectrum is not applicable and a new formalism needs to be developed to determine the effect of short range interactions of the inflaton on the power spectrum and the CMB anisotropy. We develop a general formalism for computing the power spectrum of curvature perturbations for such non-canonical cases by using the flat space Källén-Lehmann spectral function in curved quasi-de Sitter space assuming implicitly that the Bunch-Davis boundary conditions enforces the inflaton mode functions to be plane wave in the short wavelength limit and a complete set of mode functions exists in quasi-de Sitter space. It is observed that the inflaton with a decay width suppresses the power at large scale while a composite inflaton's power spectrum oscillates at large scales. These observations may be vindicated in the WMAP data and confirmed by future observations with PLANCK.

Keywords: CMBR theory, Inflation, Quantum Field Theory on curved space.

Contents

1. Introduction

In the generic inflation model [1], inflation is caused by a slow roll of the inflaton scalar field and the perturbations of the inflaton field give rise to density perturbations [2] and CMB anisotropies observed at cosmological scales. The two-point correlation function of the inflaton perturbation during inflation or the power spectrum of this two-point correlation in momentum space determines the CMB anisotropy of the universe at last scattering which we observe today. The generic slow roll model of inflation are characterized by the inflaton potential and its derivatives and a large variety of particle physics potential have been studied [3]. Inflation may be caused by more than one scalar field and these multifield models have interesting consequences in the CMB anisotropy like isocurvature perturbation [4] or large non-gaussianity in curvaton models [5]. In models of inflation with elementary scalar fields, the perturbations of the inflaton obey the Klein-Gordon equation in the quasi de Sitter space [6], whose solutions are used for calculation of the two point correlation and the curvature power spectrum.

However, it may be possible that the inflaton field is a composite of fermions and we can ask if the compositeness changes the perturbation spectrum which can be observed in the CMB anisotropy. Similarly if the inflaton is unstable with a decay width $\Gamma \sim H/N$ (such that the inflaton decays after N e-foldings of inflation are over) then again we can ask if the decay of the inflaton is reflected in the power spectrum and CMB anisotropy. For such situations the standard methods of calculating the power spectrum do not work as not all

forms of the short range structure of the scalar field are reflected in the inflaton potential. If the length scale of the scalar perturbation is of the same order as the compositeness scale then the effective theory description of the scalar potential breaks down. Similarly if the inflaton is a resonance with a lifetime of the same order as the duration of inflation $\tau \sim N/H$ i.e a width $\Gamma \sim H/N$ then there are corrections to the two point correlation that are not reflected in the inflaton potential.

In this paper we find a general method for computing the power spectrum of inflaton perturbations if the inflaton has non-trivial interactions like a decay width or if the inflaton is not an elementary scalar but a composite of fermions. *We show in general that the two-point correlations of the interacting field can be written in terms of the two-point function of the free field (in the de Sitter background) by use of the Källén-Lehmann spectral function [7, 8] if the short wavelength limit of the mode functions are plane wave states* $\frac{1}{\sqrt{2k}}e^{-ik\eta}$, *which in the quasi de-Sitter space is enforced by the assumption of the Bunch-Davis boundary conditions and there exists a complete orthonormal set of mode functions of the free theory in curved spacetime which is true in the quasi de-Sitter space relevant for inflation power spectrum calculation.* The two point correlation of an interacting theory can be written as a convolution of the free field correlation function with a spectral function $\rho(\sigma^2)$

$$G^{(\mathrm{int})}(p) = \int_0^\infty d\sigma^2 \rho(\sigma^2) G^0(p,\sigma^2), \tag{1.1}$$

where $G^{(\mathrm{int})}(p)$ is the two point correlation of the interacting theory and $G^0(p,\sigma^2)$ is the free-field correlation with mass parameter σ. The Källén-Lehmann representation holds for all two point correlations like the Feynman propagator $\Delta(p,\sigma^2)$ or the equal time Wightman function $W_{\mathrm{ET}}(x-y)$. As mentioned above we show that this result can be generalized to the curved space if we assume that a complete orthogonal basis set of states of the interacting theory exists in curved spacetime.

The power spectrum of the inflaton perturbation is related to the equal-time Wightman function in the de-Sitter space as

$$W_{\mathrm{ET}}^{\mathrm{dS}}(x) = \langle 0|(\delta\phi(\mathbf{x},t))^2|0\rangle = \int \frac{dk}{k}\mathcal{P}_{\delta\phi}(k). \tag{1.2}$$

The Bunch-Davies boundary condition is that the inflaton perturbations, in the limit where the momentum k is large compared to the inflaton horizon (for a spatially flat de-Sitter space), tend to the free field form $\delta\phi(k,\eta) = \frac{1}{\sqrt{2k}}e^{-ik\eta}$ (where η is the conformal time). Assuming the Bunch-Davies boundary conditions, if we have short range interactions which dominate at scales smaller than the inflation horizon, we may be justified in using the flat space form of the spectral function in Eq. (1.2) to compute the two point correlation function for interacting theory. Therefore power spectrum of the interacting scalar field can be expressed as

$$P^{(\mathrm{int})}(k) = \int_0^\infty P^{(0)}(k,\sigma^2)\rho\left(\sigma^2\right) d\sigma^2, \tag{1.3}$$

where $P^{(0)}(k,\sigma^2)$ is the power spectrum of the free scalar field with a mass parameter σ and $\rho(\sigma^2)$ is the KL spectral function which encapsulates all the short distance interactions (like compositeness or resonance) of the scalar field.

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In Sec. (2) we derive the relation between the power spectrum of interacting theory and the free field theory given in Eq. (1.3). In Sec. (3) we apply this result to calculate the power spectrum for case of a decaying inflaton field. We find that the TT angular spectrum of CMB anisotropy is suppressed at low l. In Sec. (4) we derive the power spectrum of the composite inflaton field. We find that the power spectrum of the composite field has a resonance which gives rise to oscillatory features in the TT angular spectrum.

2. Power spectrum of interacting scalar field - general case

The power spectrum for the inflaton is essentially given by the equal-time Wightman function in de-Sitter space. In this section we will provide a general formalism of calculating power spectrum for interacting scalar field using KL representation. Derivations of the two point correlation functions for interacting real scalar field using KL representation in Minkowski space is given in Appendix (A).

It is assumed in the following derivation that the asymptotic 'in' and 'out' states of an interacting scalar field are free particle states in the curved space. We assume the interactions being short ranged dominate over curvature effects at short distances. Since we assume the Bunch-Davies boundary conditions that the curved space mode functions in the large momentum limit go over to the flat space plane-wave form, we can directly use the flat-space calculation of spectral function of the interaction theory in the inflation power spectrum formula.

To generalize the KL formalism in de-Sitter space it is to be noted that in de-Sitter space there is no translational invariance in the time direction like Minkowski space. So, the mode functions given in Eq. (A.5) can be written in a more general form for the inflaton fluctuations as

$$\langle 0|\delta\phi(x)|n\rangle = \left(\sqrt{2p_n^0}\right)\delta\phi(p_n^0,\eta)\mathrm{e}^{i\mathbf{p}_n\cdot\mathbf{x}}\langle 0|\delta\phi(0)|n\rangle, \tag{2.1}$$

where $\delta\phi(p_n^0,\eta)$ are the free field mode functions which obey the Klein-Gordon equation in the curved background and in the flat space limit $\delta\phi(p_n^0,\eta) = \frac{1}{\sqrt{2p_n^0}}\exp\left(-ip_n^0\eta\right)$. In Appendix (B) we derive the explicit form for the mode functions of a massive scalar in the de-Sitter space.

Hence the Wightman function in de-Sitter space can be written as

$$W_{\mathrm{ET}}^{\mathrm{dS}}(x,y) = \langle 0|\delta\phi(x)\delta\phi(y)|0\rangle = \sum_n \left(2p_n^0\right)\delta\phi(p_n^0,\eta)\delta\phi(p_n^0,\eta')\mathrm{e}^{i\mathbf{p}_n\cdot(\mathbf{x}-\mathbf{y})}|\langle 0|\delta\phi(0)|n\rangle|^2 \tag{2.2}$$

Here $\langle 0|\delta\phi(0)|n\rangle$ represents the short range interactions of the interacting inflaton perturbations and according to our previous assumption can be replaced by the spectral function $\rho(q^2)$ of Minkowski space defined in Eq. (A.7) as

$$\theta(q^0)\rho(q^2) = (2\pi)^3\sum_n \delta^4(q-p_n)|\langle 0|\Phi(0)|n\rangle|^2. \tag{2.3}$$

With this definition of spectral function Eq. (2.2) can be written as

$$
\begin{aligned}
\langle 0|\delta\phi(x)\delta\phi(y)|0\rangle &= \int \frac{d^4q}{(2\pi)^3}\int_0^\infty d\sigma^2 \left(2q^0\right)\delta\phi(q^0,\eta)\delta\phi(q^0,\eta')\mathrm{e}^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}\theta(q^0)\rho(\sigma^2)\delta(q^2+\sigma^2) \\
&= \int_0^\infty d\sigma^2\rho(\sigma^2)\int \frac{d^3q}{(2\pi)^3}\delta\phi(\omega,\eta)\delta\phi(\omega,\eta')\mathrm{e}^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}. \qquad (2.4)
\end{aligned}
$$

The equivalent form of Wightman function in Minkowski space of the above equation is given in Eq. (A.19). Here $\omega = \sqrt{\mathbf{q}^2+\sigma^2}$ and in de-Sitter space $\delta\phi(\omega,\eta)$ has the solution given in Eq (B.9) with mass m of the inflaton field is replaced by the mass parameter σ. The solution for light scalar field given in Eq. (B.10) will be used in the following derivation because for very massive fields ($m_\phi > H$) the power spectrum is highly damped in superhorizon scales as given in Eq. (B.11) and hence the upper limit of σ^2 integration in the above equation should have a cut-off at m_0^2 where $m_0 \ll H$.

The equal-time Wightman function in de-Sitter space $W_{\mathrm{ET}}^{\mathrm{dS}}(x)$ gives the power spectrum for the inflaton fluctuations

$$
\begin{aligned}
\langle 0|\left(\delta\phi(x)\right)^2|0\rangle &= \int_0^{m_0^2} d\sigma^2\rho(\sigma^2)\int \frac{dq}{q}\frac{q^3}{2\pi^2}\left|\delta\phi(\omega,\eta)\right|^2 \\
&= \int \frac{dq}{q}\int_0^{m_0^2} d\sigma^2\rho(\sigma^2)\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2), \qquad (2.5)
\end{aligned}
$$

where $\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2)$ is the power spectrum of the free inflaton field given by Eq. (B.10) with m replaced by σ

$$
\mathcal{P}_{\delta\phi}^{(0)}(q,\sigma^2) = \frac{H^2}{4\pi^2}\left(\frac{q}{2aH}\right)^{\frac{2}{3}\frac{\sigma^2}{H^2}}. \qquad (2.6)
$$

Following Eq. (B.4) the power spectrum for the interacting scalar field is given by

$$
\langle 0|(\delta\phi(\mathbf{x},t))^2|0\rangle = \int \frac{dq}{q}\mathcal{P}_{\delta\phi}^{(\mathrm{int})}(q). \qquad (2.7)
$$

From Eq. (2.5) and Eq. (2.7) we get

$$
\mathcal{P}_{\delta\phi}^{(\mathrm{int})}(k) = \int_0^{m_0^2} d\sigma^2\rho(\sigma^2)\mathcal{P}_{\delta\phi}^{(0)}(k,\sigma^2), \qquad (2.8)
$$

and hence the curvature power spectrum (defined in Eq. (B.14)) for interacting inflaton field will be

$$
\mathcal{P}_{\mathcal{R}}(k) = \frac{H^2}{\dot{\phi}^2}\mathcal{P}_{\delta\phi}^{(\mathrm{int})}(k) = \frac{1}{2m_{\mathrm{Pl}}^2\epsilon}\int_0^{m_0^2}\mathcal{P}_{\delta\phi}^{(0)}(k,\sigma^2)\rho\left(\sigma^2\right)d\sigma^2, \qquad (2.9)
$$

where ϵ is the slow roll parameter of the inflaton. This form of curvature spectrum will be used as input in CAMB [9] or CMBFAST [10] to determine the CMB anisotropy spectrum from a given model of inflaton interactions.

3. Inflaton with a decay width

From the fact that the inflation must end in reheating we expect that the inflaton has couplings to other particles and it can decay into lighter particles. The inflaton decay width must be smaller than H/N (where $N \simeq 100$ is the number of e-foldings needed to solve the horizon and flatness problems). Since $\Gamma \lesssim 10^{-2}H$, the decay width term is negligible compared to the $H\delta\dot{\phi}$ term in the Klein-Gordon equation given in Eq. (B.2).

To compute the power spectrum of the decaying inflaton, we start with the Breit-Wigner propagator in flat space, of an unstable scalar particle with decay width Γ and mass m

$$\Delta^{(\mathrm{int})}(q^2) = \frac{1}{q^2 - m^2 + im\Gamma}, \tag{3.1}$$

whose spectral function has the form [11],

$$\rho(\sigma^2) = \frac{1}{\pi}\frac{m\Gamma}{(\sigma^2 - m^2)^2 + m^2\Gamma^2}. \tag{3.2}$$

Using the spectral function from Eq (3.2) in Eq (2.9) the power spectrum for inflaton with a decay width will be

$$\begin{aligned}\mathcal{P}_{\mathcal{R}}(k) = {} & \frac{H^2}{8m_{\mathrm{Pl}}^2\epsilon\pi^2}\left[\tan^{-1}\left(\frac{m}{\Gamma}\right) - \tan^{-1}\left(\frac{m^2 - m_0^2}{m\Gamma}\right)\right] + \frac{m^2}{12m_{\mathrm{Pl}}^2\epsilon\pi^2}\ln\left(\frac{z}{2}\right) \\ & \times\left[\tan^{-1}\left(\frac{m}{\Gamma}\right) - \cot^{-1}\left(\frac{m\Gamma}{m^2 - m_0^2}\right) + \frac{\Gamma}{2m}\ln\left(\frac{\left(m^2 - m_0^2\right)^2 + m^2\Gamma^2}{m^2\left(m^2 + \Gamma^2\right)}\right)\right],\end{aligned} \tag{3.3}$$

where $z = \frac{k}{aH}$ and $m_0 \ll H$ is the cut-off scale for the mass parameter σ.

In Fig (1) we plot $\mathcal{P}_{\mathcal{R}}(k)$ vs. k plot for the decaying inflaton. We observe that for the free scalar field (i.e. $\Gamma = 0$) the curvature power spectrum is scale-invariant where for the decaying inflaton the power spectrum gets suppressed at low k and increases at high k with respect to the free inflaton case. We also observe that the higher the decay width Γ, more is the suppression of power at low k and increase of power at high k.

In Fig (2) we plot the TT angular spectrum for the inflaton with a decay width. The parameters used for the above plots are $H = 10^{13}$ GeV, $m = 3.5\times10^{12}$ GeV, $m_0 = 7.5\times10^{12}$ GeV and for $\Gamma = 1.0 \times 10^{11}$ GeV, $\Gamma = 1.0 \times 10^{12}$ GeV and $\Gamma = 3.0 \times 10^{12}$ GeV the values of ϵ used are 1.412×10^{-5}, 1.29×10^{-5} and 1.069×10^{-5} respectively. In previous figure, we find that as the inflaton decay width Γ is increased the power at large distance scales gets suppressed. This results in suppression of the TT spectrum at low l with increasing decay width in this plot. A decay width of the inflaton may be a viable explanation of the WMAP observation of suppression in the TT power spectrum [12, 13].

4. Inflaton as Composite Particle

An interesting model of inflation can be with the inflaton as a GUT scale techni-pion which arises from a condensate of fermions in a GUT scale SU(N) techni-colour model [14] or the inflaton can be a composite of heavy right handed neutrinos [15]. In such models one may

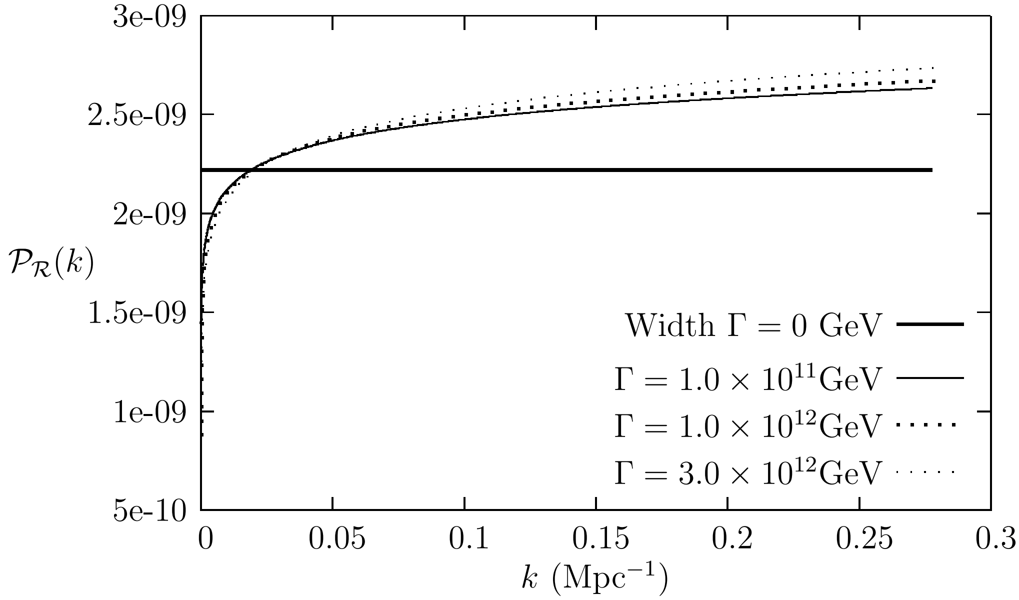

Figure 1: $\mathcal{P}_{\mathcal{R}}(k)$ vs. k plot for decaying scalar inflaton

ask in what way the compositeness of the inflaton affects the power spectrum. We use the spectral representation of a composite scalar in deriving the power spectrum.

The spectral function for a composite scalar can be taken as in QCD models [16] as

$$\rho(\sigma^2) = Z\delta(\sigma^2 - m_\varphi^2) + \frac{1}{f_\varphi^2 m_\varphi^2}\rho_c(\sigma^2)\theta(\sigma^2 - s_0^2), \tag{4.1}$$

where m_φ is the techni-pion mass, f_φ is the symmetry breaking scale and s_0 is the threshold for the onset of a continuum contribution $\rho_c(\sigma^2)$.

The wave function renormalization constant Z can be determined using the following property of the spectral function

$$\int_0^\infty \rho(\sigma^2) d\sigma^2 = 1. \tag{4.2}$$

The spectral function for the continuum is given as [17]

$$\rho_c(\sigma^2) = \frac{N}{8\pi^2}\sigma^2\left(1 - \frac{s_0^2}{\sigma^2}\right)^{\frac{3}{2}}, \tag{4.3}$$

where N is the number of fermion flavours. Using Eq. (4.1), Eq. (4.2) and Eq. (4.3) we get

$$Z = 1 - \frac{N}{8\pi^2}\frac{1}{f_\varphi^2 m_\varphi^2}\left(\frac{1}{2}\Lambda^4 - \frac{3s_0^2}{2}\Lambda^2 + s_0^4\right), \tag{4.4}$$

where Λ is the ultra-violet cut-off of the composite theory.

Now using Eq. (4.1) and Eq. (4.3) in Eq. (2.9) we find the power spectrum for a composite scalar particle as

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{ZH^2}{8\pi^2 m_{\rm Pl}^2\epsilon}\left(\frac{z}{2}\right)^{\frac{2}{3}\frac{m_\varphi^2}{H^2}} + \frac{3NH^4}{256\pi^4 m_{\rm Pl}^2\epsilon\left[\ln\left(\frac{z}{2}\right)\right]^2}\frac{1}{f_\varphi^2 m_\varphi^2}\left(\frac{z}{2}\right)^{\frac{2}{3}\frac{s_0^2}{H^2}}\left[3H^2 + s_0^2\ln\left(\frac{z}{2}\right)\right]$$

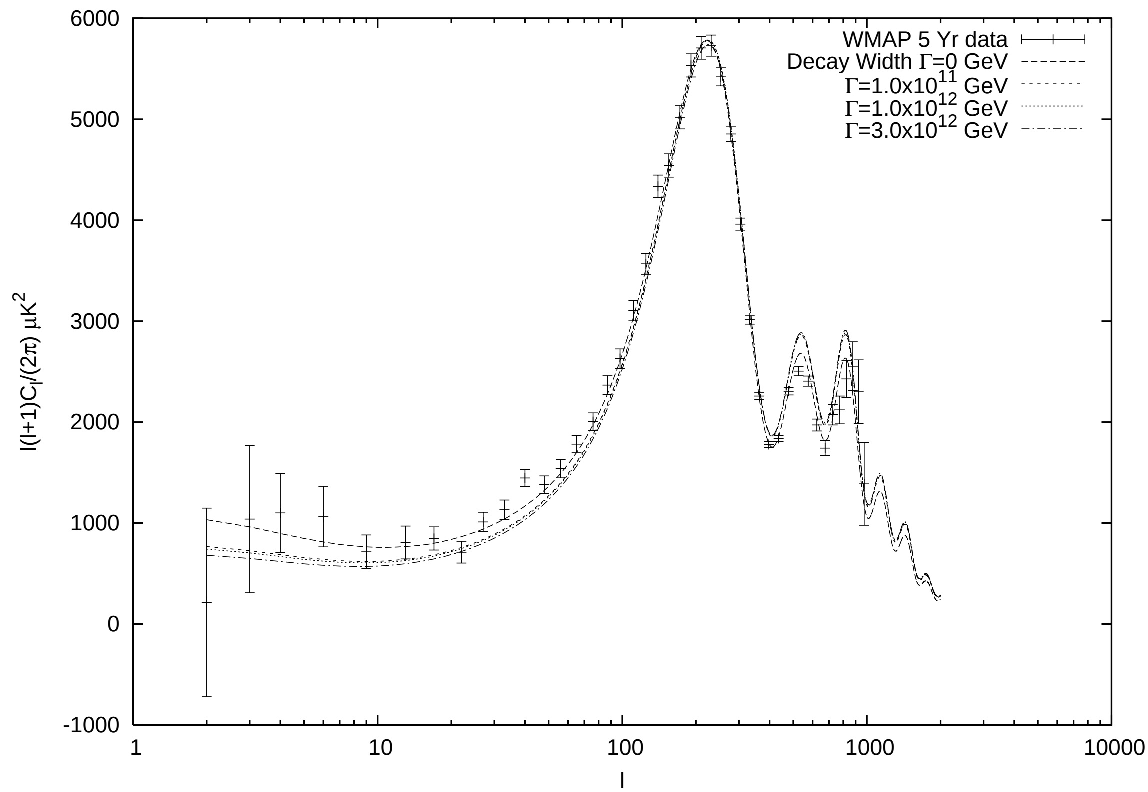

Figure 2: The TT angular spectrum for the inflaton with a decay width.

$$+\frac{3NH^4}{256\pi^4 m_{\rm Pl}^2\epsilon\left[\ln\left(\frac{z}{2}\right)\right]^2}\frac{1}{f_\varphi^2 m_\varphi^2}\left(\frac{z}{2}\right)^{\frac{2}{3}\frac{m_0^2}{H^2}}\left[-3H^2+\left(2m_0^2-3s_0^2\right)\ln\left(\frac{z}{2}\right)\right]. \tag{4.5}$$

In Fig. (3) $\mathcal{P}_{\mathcal{R}}(k)$ vs k plot for composite inflaton is given. We see that though the curvature power is scale invariant for a free scalar field (i.e. $1-Z=0$), there is a sharp resonance at $k=0.002$ Mpc^{-1}, due to compositeness ($1-Z>0$) in the inflaton field. The resonances increases as the compositeness of the inflaton increases (smaller Z). Such resonances in curvature power spectrum can lead to oscillatory features in TT angular power of CMBR as seen in other examples where spikes in the power spectrum can arise due a period of fast roll [19] or a bump in the potential [20].

In Fig (4) we plot the TT angular spectrum for the case of a composite inflaton. The parameters used for these plots are $H=10^{13}$ GeV, $m_\varphi=1.0\times10^{12}$ GeV, $m_0=3.0\times10^{12}$ GeV, $s_0=1.0\times10^{11}$ GeV, $\Lambda=1.0\times10^{13}$ GeV, $N=3$ and for $1-Z=9.7\times10^{-7}$ and $1-Z=2.0\times10^{-6}$ we take $f_\varphi=1.4\times10^{16}$ GeV, $\epsilon=3.92\times10^{-6}$ and $f_\varphi=1.0\times10^{16}$ GeV, $\epsilon=3.87\times10^{-6}$ respectively. We find that there are oscillatory features in the power spectrum at $l=30$.

Analysis of WMAP data by several groups [12] suggests that the power spectrum may have such oscillatory features. We have given the plot for some plausible values of the

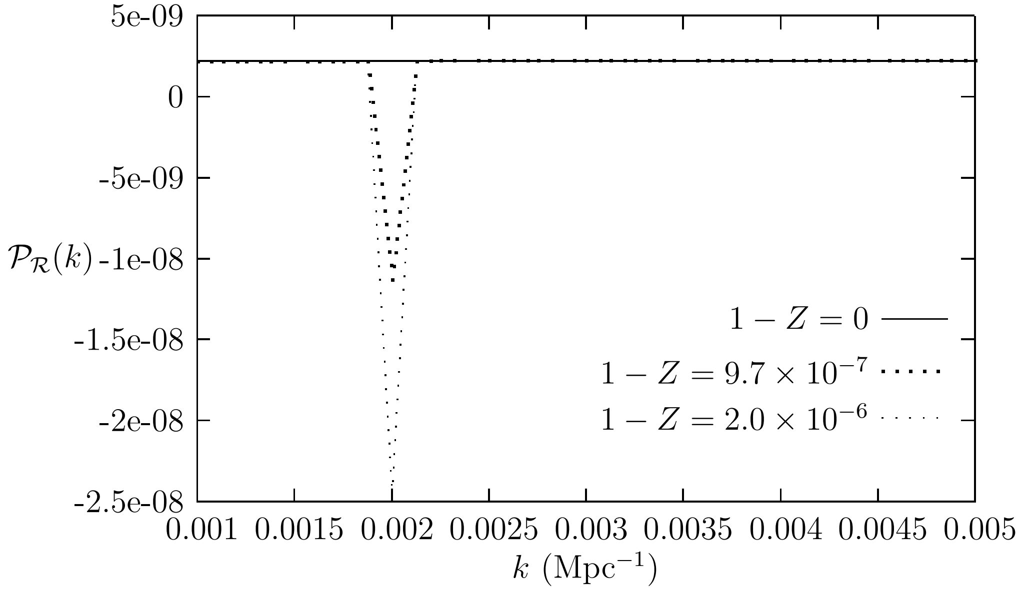

Figure 3: $\mathcal{P}_{\mathcal{R}}(k)$ vs k plot for composite inflaton

parameters. A detailed fit of the parameters with WMAP data using COSMO-MC [18] will be followed up in a forth-coming paper.

5. Conclusion

We have derived a general formula for incorporating short-range interactions in the two-point correlation functions and the power spectrum by use of the Källén-Lehmann spectral function of flat spacetime. This method is useful if the short wavelength limit of the mode functions are plane wave states $\frac{1}{\sqrt{2k}}e^{-ik\eta}$, follows in the quasi de-Sitter inflation by the assumption of the Bunch-Davis boundary conditions and there exists a complete orthonormal set of mode functions of the free theory in curved spacetime which is true in the quasi de-Sitter space relevant for inflation power spectrum calculation. In interacting inflaton models like the ones studied in this paper we find that there are more interesting variations in the power spectrum due to the modification of the propagators which do not affect the slow roll parameters. We apply our formulation to study inflation with decaying and composite inflatons. We find that the decay of the inflaton results in the suppression of long distance correlations and thereby a loss of the quadrupole anisotropy [13]. This may be related to the observation of low quadrupole power by WMAP [21].

When the inflaton is taken as a composite of two fermions the power spectrum displays even more interesting features like oscillations. An examination of the WMAP data by wavelet analysis and by the cosmic inversion method reveals that the data may have such features [12].

Acknowledgments

We have used the public domain code CAMB [9] to generate the C_l plots in Fig. (2) and

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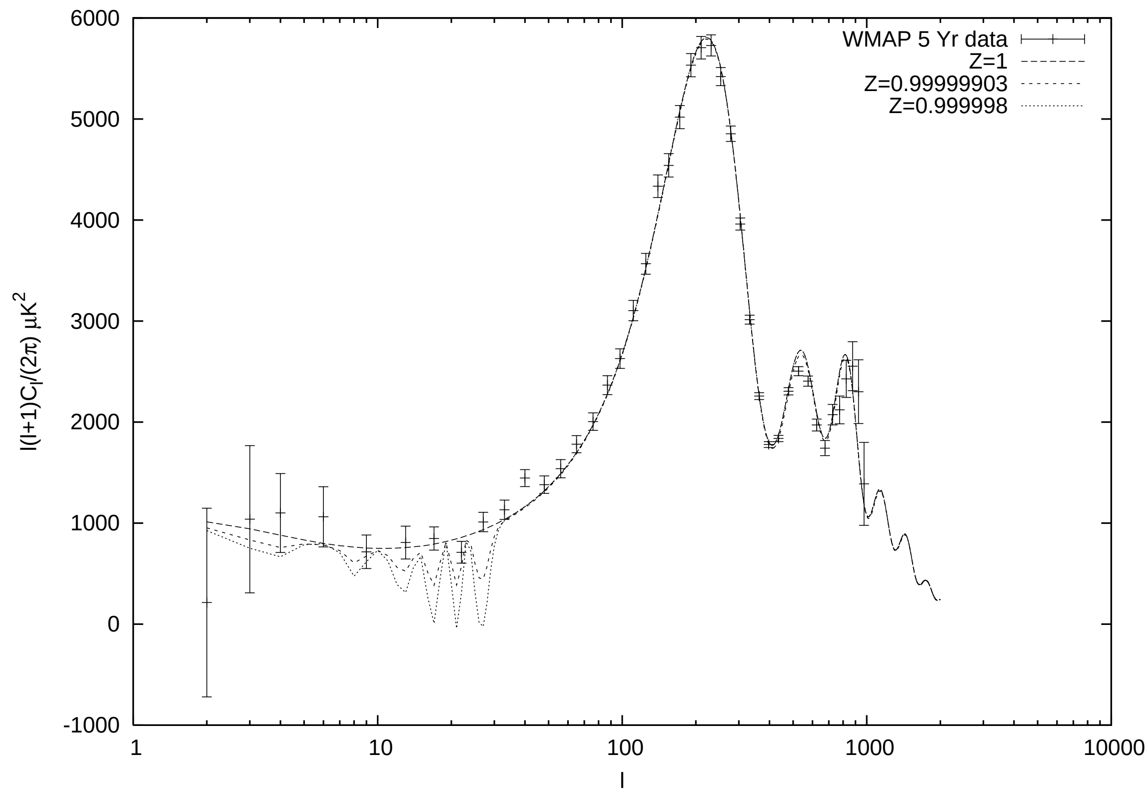

Figure 4: The TT angular spectrum for the inflaton as a composite particle.

Fig. (4). We like to thank the anonymous referee for his useful suggestions.

A. Propagator for interacting scalar field in Minkowski space using KL representation

KL representation is a non-perturbative way to derive propagator for interacting fields. Here a brief description of deriving the Feynman propagator and the Wightman function of interacting real scalar fields is being discussed. A more detailed derivation can be found in [22]. Considering a generic scalar field $\Phi(x)$, the vacuum expectation value of the time-ordered product $\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle$ gives the complete Feynman propagator for the scalar field in Fourier space

$$-i\Delta'(p) = \int d^4x \exp[ip\cdot(x-y)]\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle, \tag{A.1}$$

while the vacuum expectation value of product of two scalar fields is known as the Wightman function

$$W'(x-y) = \langle 0|\Phi(x)\Phi(y)|0\rangle. \tag{A.2}$$

Inserting a complete set of momentum eigenstates in between the two field operators the vacuum expectation value of $\Phi(x)\Phi(y)$ can be written as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \sum_n \langle 0|\Phi(x)|n\rangle\langle n|\Phi(y)|0\rangle. \tag{A.3}$$

Translational invariance in Minkowski space yields

$$\Phi(x) = \exp(ip\cdot x)\Phi(0)\exp(-ip\cdot x), \tag{A.4}$$

where

$$\begin{aligned} \langle 0|\Phi(x)|n\rangle &= \exp(-ip_n\cdot x)\langle 0|\Phi(0)|n\rangle \\ \langle n|\Phi(y)|0\rangle &= \exp(ip_n\cdot y)\langle n|\Phi(0)|0\rangle. \end{aligned} \tag{A.5}$$

In Minkowski space therefore Eq. (A.3) can be written as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \sum_n \exp(-ip_n\cdot(x-y))|\langle 0|\Phi(0)|n\rangle|^2. \tag{A.6}$$

$|\langle 0|\Phi(0)|n\rangle|^2$ encapsulating the interacting features of the scalar field can be replaced by a spectral function $\rho(q^2)$ defined as

$$\theta(q^0)\rho(q^2) = (2\pi)^3\sum_n \delta^4(q-p_n)|\langle 0|\Phi(0)|n\rangle|^2. \tag{A.7}$$

The spectral function $\rho(q^2)$ is a function of q^2 due to Lorentz invariance and is real, positive and vanishes for $q^2 < 0$. With this definition of spectral function Eq. (A.3) can be expressed as

$$\langle 0|\Phi(x)\Phi(y)|0\rangle = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(x-y;\sigma^2), \tag{A.8}$$

where

$$\Delta(x-y;\sigma^2) = \frac{1}{(2\pi)^3}\int d^4q\exp[-iq\cdot(x-y)]\theta(q^0)\delta(q^2-\sigma^2), \tag{A.9}$$

and σ is known as the mass parameter. Similarly one can find

$$\langle 0|\Phi(y)\Phi(x)|0\rangle = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(y-x;\sigma^2), \tag{A.10}$$

where

$$\Delta(y-x;\sigma^2) = \frac{1}{(2\pi)^3}\int d^4q\exp[-iq\cdot(y-x)]\theta(q^0)\delta(q^2+\sigma^2). \tag{A.11}$$

A.1 Feynman propagator for interacting scalar field

The vacuum expectation value of two time-ordered field operators is

$$\langle 0|\mathcal{T}\{\Phi(x)\Phi(y)\}|0\rangle = \Theta(x_0 - y_0)\langle 0|\Phi(x)\Phi(y)|0\rangle + \Theta(y_0 - x_0)\langle 0|\Phi(y)\Phi(x)|0\rangle. \quad \text{(A.12)}$$

Inserting Eq. (A.12), Eq. (A.8) and Eq. (A.10) in Eq. (A.1) gives the propagator for interacting scalar field as

$$-i\Delta^{(\text{int})}(p) = -i\int d^4x \exp[ip\cdot(x-y)]\int_0^\infty d\sigma^2\rho(\sigma^2)\Delta_F(x-y;\sigma^2), \quad \text{(A.13)}$$

where the Feynman propagator $\Delta_F(x-y;\sigma^2)$ for the scalar field is

$$\begin{aligned} -i\Delta_F(x-y;\sigma^2) &= \Theta(x_0-y_0)\Delta(x-y;\sigma^2) + \Theta(y_0-x_0)\Delta(y-x;\sigma^2) \\ &= \frac{-i}{(2\pi)^4}\int d^4q \exp[-iq\cdot(x-y)]\frac{1}{q^2-\sigma^2-i\varepsilon}. \end{aligned} \quad \text{(A.14)}$$

To derive the last equality the form of the step function

$$\Theta(t) = -\frac{1}{2\pi i}\lim_{\varepsilon\to 0}\int_{-\infty}^{+\infty}\frac{e^{-ist}}{s+i\varepsilon}ds \quad \text{(A.15)}$$

is to be used. This yields the form of the full propagator for the interacting scalar field in terms of the spectral function as

$$\Delta^{(\text{int})}(p) = \int_0^\infty d\sigma^2\rho(\sigma^2)\frac{1}{p^2-\sigma^2+i\varepsilon}. \quad \text{(A.16)}$$

$\frac{1}{p^2-\sigma^2+i\varepsilon}$ can be recognized as the propagator for a free scalar field with the mass m of the scalar field replaced by the mass parameter σ. Hence one can write the above equation as

$$\Delta^{(\text{int})}(p) = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta^0(p;\sigma^2), \quad \text{(A.17)}$$

where $\Delta^0(p;\sigma^2) \equiv \frac{1}{p^2-\sigma^2+i\varepsilon}$ is the free propagator of the scalar field.

A.2 Wightman function for interacting scalar field

For a free scalar field with mass m the Wightman function defined in Eq. (A.2) is

$$W^0(x-y) = \frac{1}{(2\pi)^3}\int\frac{d^3k}{2\omega_k}e^{-i\omega_k(x_0-y_0)+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}. \quad \text{(A.18)}$$

where $\omega_k \equiv \sqrt{\mathbf{k}^2+m^2}$. For the interacting scalar field the Wightman function can be derived using Eq. (A.8) which turns out to be

$$W^{(\text{int})}(x-y) = \int_0^\infty d\sigma^2\rho(\sigma^2)\Delta(x-y;\sigma^2), \quad \text{(A.19)}$$

where $\Delta(x-y;\sigma^2)$ given in Eq. (A.9) can be written as

$$\Delta(x-y;\sigma^2) = \frac{1}{(2\pi)^3}\int\frac{d^3q}{2\omega_q}e^{-i\omega_q(x_0-y_0)+i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}. \quad \text{(A.20)}$$

This can be identified as the Wightman function for the free scalar field given in Eq. (A.18) where the mass m of the scalar field is replaced by the mass parameter σ and $\omega_q \equiv \sqrt{\mathbf{q}^2+\sigma^2}$ and hence Eq. (A.19) can be written as

$$W^{(\mathrm{int})}(x-y) = \int_0^\infty d\sigma^2 \rho(\sigma^2) W^0(x-y;\sigma^2). \tag{A.21}$$

The equal-time Wightman function ($x_0 = y_0$) for the interacting scalar field has the form

$$\begin{aligned} W^{(\mathrm{int})}_{\mathrm{ET}}(x-y) &= \frac{1}{(2\pi)^3}\int_0^\infty d\sigma^2 \rho(\sigma^2) \int \frac{d^3q}{2\omega_q} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})} \\ &= \int_0^\infty d\sigma^2 \rho(\sigma^2) W^0_{\mathrm{ET}}(x-y;\sigma^2). \end{aligned} \tag{A.22}$$

B. Power spectrum of free scalar field

The useful quantity to characterize the properties of quantum fluctuations in the inflaton field is the power spectrum which is the variance (two-point correlation function) of these fluctuations. The quantum fluctuations of inflaton field cab be expanded in Fourier modes as

$$\delta\phi(\mathbf{x},t) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} e^{i\mathbf{k}\cdot\mathbf{x}} \delta\phi_{\mathbf{k}}(t), \tag{B.1}$$

which satisfy the Klein-Gordon equation in momentum space

$$\delta\ddot{\phi}_{\mathbf{k}} + 3H\delta\dot{\phi}_{\mathbf{k}} + \left(\frac{k^2}{a^2} + m_\phi^2\right)\delta\phi_{\mathbf{k}} = 0, \tag{B.2}$$

where dot represents derivative with respect to cosmic time t. The power spectrum for these fluctuations is defined as

$$\delta^3(\mathbf{k}_1 - \mathbf{k}_2)\mathcal{P}^{(0)}_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2}\langle 0|\delta\phi_{\mathbf{k}_1}\delta\phi_{\mathbf{k}_2}|0\rangle, \tag{B.3}$$

and hence the variance of the perturbations at equal time in term of power spectrum turns out to be

$$\langle 0|(\delta\phi(\mathbf{x},t))^2|0\rangle = \int \frac{dk}{k}\mathcal{P}^{(0)}_{\delta\phi}(k). \tag{B.4}$$

Defining $\delta\phi_{\mathbf{k}} = \frac{\delta\chi_{\mathbf{k}}}{a(\eta)}$, the equation of motion satisfied by $\delta\chi_{\mathbf{k}}$ can be derived from Eq. (B.2) as

$$\delta\chi''_{\mathbf{k}} + \left(k^2 - \frac{1}{\eta^2}\left(\nu_\phi^2 - \frac{1}{4}\right)\right)\delta\chi_{\mathbf{k}} = 0, \tag{B.5}$$

where the prime denotes derivative with respect to conformal time η and $\nu_\phi^2 \equiv \left(\frac{9}{4} - \frac{m_\phi^2}{H^2}\right)$. This equation for real ν_ϕ (i.e. $\frac{m_\phi}{H} < \frac{3}{2}$) has the following solution

$$\delta\chi_{\mathbf{k}} = \sqrt{-\eta}\left[c_1(k)\mathrm{H}^{(1)}_{\nu_\phi}(-k\eta) + c_2(k)\mathrm{H}^{(2)}_{\nu_\phi}(-k\eta)\right], \tag{B.6}$$

where $\mathrm{H}^{(1)}_{\nu_\phi}$ and $\mathrm{H}^{(2)}_{\nu_\phi}$ are the Hankel functions of the first and second kind respectively having the form

$$\mathrm{H}^{(1)}_{\nu_\phi}(x \gg 1) \sim \sqrt{\frac{2}{\pi x}} e^{i\left(x - \frac{\pi}{2}\nu_\phi - \frac{\pi}{4}\right)}$$
$$\mathrm{H}^{(2)}_{\nu_\phi}(x \gg 1) \sim \sqrt{\frac{2}{\pi x}} e^{-i\left(x - \frac{\pi}{2}\nu_\phi - \frac{\pi}{4}\right)}. \tag{B.7}$$

Imposing that the modes of these scalar fluctuations inside the Hubble radius ($k \gg aH$) have the plane wave solution $\frac{e^{-ik\eta}}{\sqrt{2k}}$ yields $c_1(k) = \frac{\sqrt{\pi}}{2} e^{i\left(\nu_\phi + \frac{1}{2}\right)\frac{\pi}{2}}$ and $c_2(k) = 0$. On super-horizon scale ($k \ll aH$) using the asymptotic behaviour of the Hankel function

$$\mathrm{H}^{(1)}_{\nu_\phi}(x \ll 1) \sim \sqrt{\frac{2}{\pi}} e^{-i\frac{\pi}{2}} 2^{\left(\nu_\phi - \frac{3}{2}\right)} \frac{\Gamma\left(\nu_\phi\right)}{\Gamma\left(\frac{3}{2}\right)} x^{-\nu_\phi}, \tag{B.8}$$

the solution for $\delta\phi_{\mathbf{k}}$ turns out to be

$$|\delta\phi_{\mathbf{k}}| \simeq \frac{H}{\sqrt{2k^3}} \left(\frac{k}{2aH}\right)^{\frac{3}{2} - \nu_\phi}, \tag{B.9}$$

and hence the power spectrum defined in Eq. (B.3) for this light scalar field ($m_\phi < H$) will be

$$\mathcal{P}^{(0)}_{\delta\phi}(k) = \frac{H^2}{4\pi^2} \left(\frac{k}{2aH}\right)^{\frac{2}{3}\frac{m_\phi^2}{H^2}}. \tag{B.10}$$

For very massive scalar field ($m_\phi > H$) i.e. for imaginary ν_ϕ the form of power spectrum can be derived as

$$\mathcal{P}^{(0)}_{\delta\phi}(k) \simeq \frac{H^2}{4\pi^2} \left(\frac{H}{m_\phi}\right) \left(\frac{k}{aH}\right)^3, \tag{B.11}$$

which is suppressed by the ratio $\left(\frac{H}{m_\phi}\right)$ and hence highly damped at large wavelengths.

These quantum fluctuations in inflaton field generates fluctuation in the metric which is coupled to it through Einstein's equation. The perturbed FRW metric has the form (considering only scalar perturbations)

$$\tilde{g}_{\mu\nu} = a^2(\eta) \begin{pmatrix} 1 + 2A & 0 \\ 0 & -(1 - 2\psi)\delta_{ij} \end{pmatrix}, \tag{B.12}$$

where the quantity ψ is known as the curvature perturbation. The gauge invariant quantity formed out of this perturbation is known as the comoving curvature perturbation and defined as

$$\mathcal{R} = \psi + H\frac{\delta\phi}{\dot{\phi}}. \tag{B.13}$$

The CMB anisotropy spectrum is determined by the power spectrum of this comoving curvature perturbation which is related to the power spectrum of scalar perturbation as

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{1}{2m_{\rm Pl}^2\epsilon}\mathcal{P}_{\delta\phi}^{(0)}(k), \tag{B.14}$$

where ϵ is the slow-roll parameter and $m_{\rm Pl} = \frac{1}{\sqrt{8\pi G}}$, G being the Newton's Gravitational constant.

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